Study on Mobile Tariff Selection based on Multinomial Logistic Model

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Keywords: Logistic model; Parameter estimation; Consistent estimates

Abstract: This paper mainly studies the problem on tariff choice of mobile company. First, the logistic selection model is established by analyzing the data information of customer's tariff selection in a mobile company. Then the maximum likelihood estimation method is used to estimate the parameters in the model. Finally, the rationality of the parameters estimated by the consistency test is given.

1. Introduction

Multinomial logistic model is an effective tool for credit risk identification of commercial banks in China, which has been widely used in reality. For example, there are k stores with q characteristics. If we want to study how customers choose stores, we can consider the multinomial logistic selection model.

According to the theoretical and empirical studies of domestic scholars over the years, Logistic model[1][2] had a very credible ability to identify, predict and promote, and it had the following probability density

$$p\left(Y=j \mid X=x\right)=\frac{\exp\left(\beta^{T} x_{j}\right)}{\sum_{t=1}^{k}\exp\left(\beta^{T} x_{t}\right)} \qquad \left(j=1,2,\cdots,k\right)$$

$X_1, X_2, \cdots, X_k$ was independent with each other.

X_j is the column vector evaluated at R^q, $X_j \sim N\left(\mu_j, \sigma_j\right)$, $\mu_j, \sigma_j > 0$ is unkown, $j=1,2,\cdots,k$. Y is a random variable whose value is $1,2,\cdots,k$. $X=\left(X_1, X_2, \cdots, X_k\right)$, x is the value of X. $\beta \in R^q$ is the unknown parameter, $\left(\|\beta\|>0\right), k>1, q>1$.

2. Modelling

With the development of communication technology, mobile phone has become an indispensable tool in people's daily life. How to choose a suitable tariff type among the numerous optional tariff launched by mobile companies is very important. This paper analyzes the data information of a mobile company's past customers' tariff selection, and the following multinomial logistic selection model is constructed

$$p(Y=j\mid X=x)=\frac{\exp(\beta^T x_j)}{\sum_{t=1}^{10}\exp(\beta^T x_t)} \qquad (j=1,2,\cdots,10)$$

$X_1, X_2, \cdots, X_{10}$ are independent with each other. X_j is a 3-dimensional column vector, $X_j \sim N(\mu_j, \sigma_j)$, $j=1,2,\cdots,10$. Y is a random variable whose value is $1,2,\cdots,10$; $X=(X_1, X_2, \cdots, X_{10})$, x is the value of X. $\beta \in R^3$ is the unknown parameter, and $\|\beta\|>0$.

3. Parameter unit vector estimation

The overall (X,Y) observation value is shown in table 1, where X represents the monthly function fee (yuan), domestic data flow (G) and duration of call (minutes) of the selling price

Tab1 Data of test

Y	1	2	3	4	5	6	7	8	9	10
X	$\begin{pmatrix}58\\0.15\\150\end{pmatrix}$	$\begin{pmatrix}88\\0.3\\350\end{pmatrix}$	$\begin{pmatrix}128\\0.6\\650\end{pmatrix}$	$\begin{pmatrix}158\\0.6\\900\end{pmatrix}$	$\begin{pmatrix}188\\0.6\\1200\end{pmatrix}$	$\begin{pmatrix}58\\0.5\\50\end{pmatrix}$	$\begin{pmatrix}88\\0.7\\200\end{pmatrix}$	$\begin{pmatrix}128\\1\\420\end{pmatrix}$	$\begin{pmatrix}158\\2\\510\end{pmatrix}$	$\begin{pmatrix}188\\2.5\\600\end{pmatrix}$
sales	94401	58899	18887	4457	5798	46713	20369	4799	1952	1426

Label the samples $(X^{(i)}, Y^{(i)})$ $(i=1,2,\cdots,257701)$ in order of sale

Let $S_d=\{i\mid Y^{(i)}=d\}$, N_d is the number of elements in S_d, $d=1,2,\cdots,10$;

$$\bar{X}=\frac{1}{257701}\sum_{i=1}^{257701} X^{(i)}=\begin{pmatrix}79.793\\0.385\\265.022\end{pmatrix}$$

Let $\hat{\mu}_d = X^{(i)}-\bar{X}$, the values are shown in table 2 and table 3.

Tab2 Data of test

d	1	2	3	4	5
$\hat{\mu}_d$	$\begin{pmatrix}-21.793\\-0.235\\-115.022\end{pmatrix}$	$\begin{pmatrix}8.207\\-0.085\\84.978\end{pmatrix}$	$\begin{pmatrix}48.207\\0.215\\384.978\end{pmatrix}$	$\begin{pmatrix}78.207\\0.215\\634.978\end{pmatrix}$	$\begin{pmatrix}108.207\\0.215\\934.978\end{pmatrix}$

Tab3 Data of test

d	6	7	8	9	10
$\hat{\mu}_d$	$\begin{pmatrix}-21.793\\0.115\\-215.022\end{pmatrix}$	$\begin{pmatrix}8.207\\0.315\\-65.022\end{pmatrix}$	$\begin{pmatrix}48.207\\0.615\\154.978\end{pmatrix}$	$\begin{pmatrix}78.207\\1.615\\244.978\end{pmatrix}$	$\begin{pmatrix}108.207\\2.115\\334.978\end{pmatrix}$

Let $\hat{V}_d = \sum_{d=1}^{10} \frac{N_d}{100} \hat{\mu}_d \hat{\mu}_d^T$; $\hat{V}_d = \begin{pmatrix} 975 & 6 & 6865 \\ 6 & 0 & 24 \\ 6865 & 24 & 54239 \end{pmatrix}$

Let $\hat{V} = \hat{\Sigma}^{-1} \hat{V}_d \hat{\Sigma}^{-1}, \hat{\Sigma} = \frac{1}{10} \sum_{d=1}^{10} \hat{\mu}_d \hat{\mu}_d^T$;

$$\hat{\Sigma} = \begin{pmatrix} 4140 & 40 & 23960 \\ 40 & 0 & 160 \\ 23960 & 160 & 169270 \end{pmatrix}; \hat{\Sigma}^{-1} = \begin{pmatrix} 0.0643 & -2.1961 & -0.007 \\ -2.1961 & 76.5520 & 0.2388 \\ -0.007 & 0.2388 & 0.0008 \end{pmatrix}$$

$$\hat{V} = \begin{pmatrix} 0.025 & -0.8479 & -0.0027 \\ -0.8479 & 28.9425 & 0.091 \\ -0.0027 & 0.091 & 0.0003 \end{pmatrix}$$

The maximum characteristic root of $\hat{V}$ is λ_1 =28.9676, its corresponding eigenvector is $\hat{\eta} = \begin{pmatrix} 0.0293 \\ -0.9996 \\ -0.0031 \end{pmatrix}$.The estimation of the unit vector $\frac{\beta}{\|\beta\|}$ of parameter β is $\hat{\beta}_d = \hat{\eta} = \begin{pmatrix} 0.0293 \\ -0.9996 \\ -0.0031 \end{pmatrix}$.

4. Estimation of parametric vector module

The maximum likelihood method [3][4] is used to estimate the module α of the parameter β.The multinomial logistic selection model constructed in this paper can be rewritten as

$$p(Y = y \mid X = x) = \frac{\exp\left(\|\beta\| \beta_d^T x_y\right)}{\sum_{t=1}^{10} \exp\left(\|\beta\| \beta_d^T x_t\right)}$$

Replace the estimator of parameter unit vector $\hat{\beta}_d$ with the estimator β_d in the above equation, then it can get the formula as follows,

$$p(Y = y \mid X = x) = \frac{\exp\left(\alpha \hat{\beta}_d^T x_y\right)}{\sum_{t=1}^{10} \exp\left(\alpha \hat{\beta}_d^T x_t\right)}$$

Starting from the above equation, the joint density of (X,Y) is

$$p(x,y) = p_{Y|X}(y)\, p_X(x)$$
$$= \frac{\exp\left(\alpha \hat{\beta}_d^T x_y\right)}{\sum_{t=1}^{10} \exp\left(\alpha \hat{\beta}_d^T x_t\right)} \prod_{j=1}^{257701} \frac{1}{\left(\sqrt{2\pi}\right)^4} \left|\Sigma_j\right|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\left(x_j - \mu_j\right)^T \Sigma_j^{-1} \left(x_j - \mu_j\right)\right]$$

Omit the irrelevant parts with α and get the likelihood function as follow,

$$L\left(\alpha; y^{(1)},\cdots,y^{(257701)}\right)=\prod_{i=1}^{257701}\frac{\exp\left(\alpha\hat{\beta}_d^T x_y\right)}{\sum_{t=1}^{10}\exp\left(\alpha\hat{\beta}_d^T x_t\right)}$$

$$=\frac{\exp\begin{pmatrix}94401\alpha\hat{\beta}_d^T x_1+58899\alpha\hat{\beta}_d^T x_2+18887\alpha\hat{\beta}_d^T x_3+4457\alpha\hat{\beta}_d^T x_4+5798\alpha\hat{\beta}_d^T x_5\\+46713\alpha\hat{\beta}_d^T x_6+20369\alpha\hat{\beta}_d^T x_7+4799\alpha\hat{\beta}_d^T x_8+1952\alpha\hat{\beta}_d^T x_9+1426\alpha\hat{\beta}_d^T x_{10}\end{pmatrix}}{\left[\sum_{t=1}^{10}\exp\left(\alpha\hat{\beta}_d^T x_t\right)\right]^{257701}}$$

The logarithmic likelihood function is

$$\begin{aligned}l\left(\alpha; y^{(1)},\cdots,y^{(257701)}\right)&=94401\alpha\hat{\beta}_d^T x_1+58899\alpha\hat{\beta}_d^T x_2+18887\alpha\hat{\beta}_d^T x_3\\&+4457\alpha\hat{\beta}_d^T x_4+5798\alpha\hat{\beta}_d^T x_5+46713\alpha\hat{\beta}_d^T x_6+20369\alpha\hat{\beta}_d^T x_7\\&+4799\alpha\hat{\beta}_d^T x_8+1952\alpha\hat{\beta}_d^T x_9+1426\alpha\hat{\beta}_d^T x_{10}-257701\ln\sum_{t=1}^{10}\exp\left(\alpha\hat{\beta}_d^T x_t\right)\end{aligned}$$

The likelihood equation is

$$\sum_{i=1}^{257701}\hat{\beta}_d^T x^{(i)}-257701\frac{\sum_{t=1}^{10}\hat{\beta}_d^T x_t\exp\left(\alpha\hat{\beta}_d^T x_t\right)}{\sum_{t=1}^{10}\exp\left(\alpha\hat{\beta}_d^T x_t\right)}=0$$

According to the calculated result, the likelihood equation can be written as

$$291610-\frac{257701\begin{pmatrix}1.0845\exp(1.0845\alpha)+1.1935\exp(1.1935\alpha)\\+1.1356\exp(1.1356\alpha)+1.2396\exp(1.2396\alpha)\\+1.1886\exp(1.1886\alpha)+1.0446\exp(1.0446\alpha)\\+1.2587\exp(1.2587\alpha)+1.4488\exp(1.4488\alpha)\\+1.0492\exp(1.0492\alpha)+1.1494\exp(1.1494\alpha)\end{pmatrix}}{\begin{array}{l}\exp(1.0845\alpha)+\exp(1.1935\alpha)+\exp(1.1356\alpha)+\exp(1.2396\alpha)\\+\exp(1.1886\alpha)+\exp(1.0446\alpha)+\exp(1.2587\alpha)+\exp(1.4488\alpha)\\+\exp(1.0492\alpha)+\exp(1.1494\alpha)\end{array}}=0$$

Merge similar items and get

$$\begin{aligned}&12133.2655\exp(1.0845\alpha)-15956.1435\exp(1.1935\alpha)-1035.2556\exp(1.1356\alpha)\\&-27836.1596\exp(1.2396\alpha)-14693.4086\exp(1.1886\alpha)+22415.5354\exp(1.0446\alpha)\\&-32758.2487\exp(1.2587\alpha)-81747.2088\exp(1.4488\alpha)+21230.1108\exp(1.0492\alpha)\\&-4591.5294\exp(1.1494\alpha)=0\end{aligned}$$

Let $A=e^{\alpha}$, the above equation can be written as

$$\begin{aligned}&12133.2655A^{1.0845}-15956.1435A^{1.1935}-1035.2556A^{1.1356}-27836.1596A^{1.2396}\\&-14693.4086A^{1.1886}+22415.5354A^{1.0446}-32758.2487A^{1.2587}\\&-81747.2088A^{1.4488}+21230.1108A^{1.0492}-4591.5294A^{1.1494}=0\end{aligned}$$

Multiply both sides of the equation with $A^{-0.4488}$, then can get the follow equation

$$12133.2655A^{1.0845} - 15956.1435A^{0.7447} - 1035.2556A^{0.6868} - 27836.1596A^{0.7908}$$
$$-14693.4086A^{0.7398} + 22415.5354A^{0.5958} - 32758.2487A^{0.8099}$$
$$+21230.1108A^{0.6004} - 4591.5294A^{0.7006} = 81747.2088A$$

Divide both sides of the equation by 81747.2088 to get

$$0.1484A^{1.0845} - 0.1952A^{0.7447} - 0.0127A^{0.6868} - 0.3405A^{0.7908} - 0.1797A^{0.7398}$$
$$+0.2742A^{0.5958} - 0.4007A^{0.8099} + 0.2597A^{0.6004} - 0.0562A^{0.7006} = A$$

Using iterative method to get A =1.0320, e^{α} =1.0320, then get $\hat{\alpha}$ =0.0315.

5. Consistency estimation of parameter vectors

According to Slutsky's theorem, the estimation of parameter vectors is consistent.

$$\hat{\beta} = \hat{\alpha} \cdot \hat{\beta}_d = 0.0315 \cdot \begin{pmatrix} 0.0293 \\ -0.9996 \\ -0.0031 \end{pmatrix} = \begin{pmatrix} 0.00092295 \\ -0.0314874 \\ -0.00009765 \end{pmatrix}$$

Acknowledgement

Project supported by the National Natural Science Foundation of China (No.11732010, 11572221,11872272) ,the Scientific Research Program Project of Tianjin Education Commission (No.2018KJ267), scientific research projects of Tianjin university Renai college(No.XX18002) and Educational teaching and new engineering research project of 2018 of Shandong huayu university of technology(No.2018XGK02)

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